Text reference page 125.

Exploring Properties of Inverses

Purpose
To find the inverse of a matrix using the algorithm from Section 2.2 of the text.

MATLAB Functions
`gauss, bgauss, scale, inv`

The algorithm described on page 124 of the text shows how to find the inverse of a matrix by row reducing the augmented matrix $[A\ I_n]$. We can do this easily in MATLAB:

```
A = magic(5)
B = [A, eye(5)]
```

First, we scale the entry in the (1, 1) position to 1:

```
B = scale(B,1/B(1,1)),
```

Next, we zero out the column below the (1,1) entry with `gauss`:

```
B = gauss(B,1)
```

Now, we pivot on the (2, 2) entry, by first scaling

```
B = scale(B,1/B(2,2))
```

then zeroing out below the pivot with `gauss`,

```
B = gauss(B,2)
```

and next zeroing out above the pivot with `bgauss`

```
B =bgauss(B,2)
```

We complete the row reduction of the matrix using `gauss, bgauss`, and `scale`, and finally, we peel out the inverse with MATLAB's colon notation:

```
B = B(:,6:12)
```

You can check that we have obtained the inverse using

```
eye(5) - A*B
```

MATLAB also has an inverse function, `inv`. Try this on the matrix given above,

```
inv(A)
```

and compare the result with the inverse we have computed

```
B - inv(A)
```

MATLAB Exercises

1. Find the inverse (if it exists) of each of the following matrices by row reducing the augmented matrix $[A\ I_n]$ with the MATLAB functions `gauss, bgauss`, and `scale`. Check your work as described above.

a. `A = magic(5)`

b. `A = pascal(5)`

c. `A = magic(6)`

2. Solve the equation $Ax = e_1$, where `A = pascal(4)`, using the Laydata functions `swap, gauss,` and `bgauss`. Then form `inv(A)`. Describe where the solution `x` appears in `inv(A)`. Now solve $Ax = e_2$ and describe where the solution appears in `inv(A)`. Expain how this is related to the algorithm on page 124 of the text.

3. Solve the equation $Ax = b$ using MATLAB's inverse function.

a. `A = magic(5), b = (1:5)'`

b. `A = vander(1:5), b = ones(5,1)`

Check your answers with `A*x - b`. Now solve $xA = b^T$ using the matrices from parts a and b and check your answers.

4. How does MATLAB respond when you try `inv(A)` for `A = magic(4)`? If `B = inv(A)`, what happens when you try `A*B` and `B*A` ? What do you conclude about `A`?

5. Let A be the matrix

$$A = \begin{bmatrix} -1 & -1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Using MATLAB

a. Show that the identity, $A^3 + A^2 + A + I_4 = 0$ holds.

b. Use this identity to solve $AX = I_4$ for X.

c. Compare the solution `X` with `inv(A)` using `X - inv(A)`.

6. Using MATLAB, solve the equation $AXB = C$, where `A = pascal(4)`, `B = vander(1:4)`, and `C = magic(4)`. Check your answer using `C - A*X*B`. Is your answer, `X`, invertible? Check this in MATLAB and then justify your answer.

7. In MATLAB, let

```
A = rand(4); A = A/(max(sum(A)) + 1)
```

We are going to look at the sum $I_4 + A + A^2 + A^3 + \cdots$. In MATLAB we can form this sum quickly as

```
S = eye(4);
for k = 1:10 S = eye(4) + A*S; end, S
```

Repeat the `for` loop a few times until the value for `S` stabilizes. Now it is reasonable to ask what is this value is.n The formula for a geometric sum when is

$$(x-1)\sum_{k=0}^{n} x^k = x^{n+1} - 1$$

When we set $x = A$, we get

$$(A - I_4)\sum_{k=0}^{n} A^k = A^{n+1} - I_4$$

What is happening to A^{n+1} as n gets large? Use MATLAB to compute A^{10}, A^{20}, etc. When we set up

```
A = A/(max(sum(A)) + 1)
```

we were ensuring that this would happen (look ahead to Theorem 11 on page 154 of the text.) So the sum $I_4 + A + A^2 + A^3 + \cdots$ should converge leaving the equation

$$(A - I_4)S = -I_4$$

What is S? Use MATLAB to verify your answer.